

6.4

A

1–13 ■ Use the Table of Integrals [on Reference Pages 6–10] to evaluate the integral.

1. $\int x \sin^{-1}(x^2) dx$

2. $\int x^3 \sin^{-1}(x^2) dx$

3. $\int \frac{\sin x \cos x}{\sqrt{1 + \sin x}} dx$

4. $\int e^x \cos(3x + 4) dx$

5. $\int e^x \ln(1 + e^x) dx$

6. $\int \frac{\sqrt{9x^2 - 1}}{x^2} dx$

7. $\int \frac{\sqrt{4 - 3x^2}}{x} dx$

8. $\int \frac{\sin \theta}{1 + 2 \cos \theta} d\theta$

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9. $\int_{-2}^1 \sqrt{5 - 4x - x^2} \, dx$

10. $\int \sqrt{2 + 3 \cos x} \tan x \, dx$

11. $\int \frac{x}{\sqrt{x^2 - 4x}} dx$

12. $\int_0^{\pi/2} \cos^5 x \, dx$

13. $\int x^2 \tan^{-1} x \, dx$

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14. Find the volume of the solid obtained when the region under the curve $y = 1/(1 + 5x)^2$ from 0 to 1 is rotated about the y -axis.

6.4 **ANSWERS****E** [Click here for exercises.](#)**S** [Click here for solutions.](#)

1. $\frac{1}{2} (x^2 \sin^{-1}(x^2) + \sqrt{1-x^4}) + C$
2. $\frac{2x^4-1}{8} \sin^{-1}(x^2) + \frac{x^2\sqrt{1-x^4}}{8} + C$
3. $-\frac{2}{3} (2 - \sin x) \sqrt{1 + \sin x} + C$
4. $\frac{3}{10} e^x \left[\frac{1}{3} \cos(3x+4) + \sin(3x+4) \right] + C$
5. $(1 + e^x) \ln(1 + e^x) - e^x + C$
6. $-\frac{\sqrt{9x^2-1}}{x} + 3 \ln |3x + \sqrt{9x^2-1}| + C$
7. $\sqrt{4-3x^2} - 2 \ln \left| \frac{2 + \sqrt{4-3x^2}}{x} \right| + C$
8. $-\frac{1}{2} \ln |1 + 2 \cos \theta| + C$
9. $\frac{9\pi}{4}$
10. $-2\sqrt{2+3\cos x} - \sqrt{2} \ln \left| \frac{\sqrt{2+3\cos x} - \sqrt{2}}{\sqrt{2+3\cos x} + \sqrt{2}} \right| + C$
11. $\sqrt{x^2-4x} + 2 \ln |x-2 + \sqrt{x^2-4x}| + C$
12. $\frac{8}{15}$
13. $\frac{1}{3} x^3 \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2) + C$
14. $\frac{2\pi}{25} (\ln 6 - \frac{5}{6})$

6.4 SOLUTIONS



1. Let $u = x^2$. Then $du = 2x dx$, so

$$\begin{aligned}\int x \sin^{-1}(x^2) dx &= \frac{1}{2} \int \sin^{-1} u du \\ &\stackrel{87}{=} \frac{1}{2} (u \sin^{-1} u + \sqrt{1-u^2}) + C \\ &= \frac{1}{2} (x^2 \sin^{-1}(x^2) + \sqrt{1-x^4}) + C\end{aligned}$$

2. Let $u = x^2$. Then $du = 2x dx$, so

$$\begin{aligned}\int x^3 \sin^{-1}(x^2) dx &= \frac{1}{2} \int u \sin^{-1} u du \\ &\stackrel{90}{=} \frac{2u^2-1}{8} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{8} + C \\ &= \frac{2x^4-1}{8} \sin^{-1}(x^2) + \frac{x^2\sqrt{1-x^4}}{8} + C\end{aligned}$$

3. Let $u = \sin x$. Then $du = \cos x dx$, so

$$\begin{aligned}\int \frac{\sin x \cos x}{\sqrt{1+\sin x}} dx &= \int \frac{u du}{\sqrt{1+u}} \stackrel{55}{=} \frac{2}{3} (u-2) \sqrt{1+u} + C \\ &= -\frac{2}{3} (2-\sin x) \sqrt{1+\sin x} + C\end{aligned}$$

4. Let $u = 3x + 4$. Then

$$\begin{aligned}\int e^x \cos(3x+4) dx &= \frac{1}{3} \int e^{(u-4)/3} \cos u du \\ &= \frac{1}{3} e^{-4/3} \int e^{u/3} \cos u du \\ &\stackrel{99}{=} \frac{1}{3} e^{-4/3} \frac{e^{u/3}}{(1/3)^2 + 1^2} \left(\frac{1}{3} \cos u + \sin u \right) + C \\ &= \frac{3}{10} e^x \left[\frac{1}{3} \cos(3x+4) + \sin(3x+4) \right] + C\end{aligned}$$

5. Let $u = 1 + e^x$, so $du = e^x dx$. Then

$$\begin{aligned}\int e^x \ln(1+e^x) dx &\stackrel{100}{=} \int \ln u du = u \ln u - u + C \\ &= (1+e^x) \ln(1+e^x) - e^x - 1 + C \\ &= (1+e^x) \ln(1+e^x) - e^x + C_1\end{aligned}$$

6. Let $u = 3x$. Then $du = 3 dx$, so

$$\begin{aligned}\int \frac{\sqrt{9x^2-1}}{x^2} dx &= \int \frac{\sqrt{u^2-1}}{u^2/9} \frac{du}{3} = 3 \int \frac{\sqrt{u^2-1}}{u^2} du \\ &\stackrel{42}{=} -\frac{3\sqrt{u^2-1}}{u} + 3 \ln |u + \sqrt{u^2-1}| + C \\ &= -\frac{\sqrt{9x^2-1}}{x} + 3 \ln |3x + \sqrt{9x^2-1}| + C\end{aligned}$$

7. By Formula 32,

$$\begin{aligned}\int \frac{\sqrt{4-3x^2}}{x} dx &= \int \frac{\sqrt{4-u^2}}{u/\sqrt{3}} \frac{du}{\sqrt{3}} \quad \left[\begin{array}{l} u = \sqrt{3}x, \\ du = \sqrt{3} dx \end{array} \right] \\ &= \int \frac{\sqrt{4-u^2}}{u} du = \sqrt{4-u^2} - 2 \ln \left| \frac{2+\sqrt{4-u^2}}{u} \right| + C_1 \\ &= \sqrt{4-3x^2} - 2 \ln \left| \frac{2+\sqrt{4-3x^2}}{\sqrt{3}x} \right| + C_1 \\ &= \sqrt{4-3x^2} - 2 \ln \left| \frac{2+\sqrt{4-3x^2}}{x} \right| + C\end{aligned}$$

$$\begin{aligned}8. \int \frac{\sin \theta}{1+2 \cos \theta} d\theta &= -\frac{1}{2} \int \frac{1}{u} du \quad \left[\begin{array}{l} u = 1+2 \cos \theta, \\ du = -2 \sin \theta d\theta \end{array} \right] \\ &= -\frac{1}{2} \ln |u| + C = -\frac{1}{2} \ln |1+2 \cos \theta| + C\end{aligned}$$

$$\begin{aligned}9. \int_{-2}^1 \sqrt{5-4x-x^2} dx &= \int_{-2}^1 \sqrt{5-(x^2+4x)} dx \\ &= \int_{-2}^1 \sqrt{5+4-(x^2+4x+4)} dx \\ &= \int_{-2}^1 \sqrt{9-(x+2)^2} dx \\ &= \int_0^3 \sqrt{3^2-u^2} du \quad [u = x+2, du = dx] \\ &\stackrel{30}{=} \left[\frac{u}{2} \sqrt{9-u^2} + \frac{9}{2} \sin^{-1} \frac{u}{3} \right]_0^3 \\ &= \left[\left(0 + \frac{9}{2} \cdot \frac{\pi}{2} \right) - (0+0) \right] = \frac{9\pi}{4}\end{aligned}$$

$$\begin{aligned}10. \int \sqrt{2+3 \cos x} \tan x dx &= -\int \frac{\sqrt{2+3 \cos x}}{\cos x} (-\sin x dx) \\ &= -\int \frac{\sqrt{2+3u}}{u} du \quad (u = \cos x, du = -\sin x dx) \\ &\stackrel{58}{=} -2\sqrt{2+3u} - 2 \int \frac{du}{u\sqrt{2+3u}} \\ &\stackrel{57}{=} -2\sqrt{2+3u} - 2 \cdot \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{2+3u}-\sqrt{2}}{\sqrt{2+3u}+\sqrt{2}} \right| + C \\ &= -2\sqrt{2+3 \cos x} - \sqrt{2} \ln \left| \frac{\sqrt{2+3 \cos x}-\sqrt{2}}{\sqrt{2+3 \cos x}+\sqrt{2}} \right| + C\end{aligned}$$

11. Let $u = x - 2$. Then

$$\begin{aligned}\int \frac{x dx}{\sqrt{x^2-4x}} &= \int \frac{(x-2)+2}{\sqrt{(x-2)^2-4}} dx \\ &= \int \frac{u du}{\sqrt{u^2-4}} + 2 \int \frac{du}{\sqrt{u^2-4}} \\ &= \frac{1}{2} \int v^{-1/2} dv + 2 \int \frac{du}{\sqrt{u^2-4}} \quad \left[\begin{array}{l} v = u^2-4, \\ dv = 2u du \end{array} \right] \\ &\stackrel{2,43}{=} v^{1/2} + 2 \ln |u + \sqrt{u^2-4}| + C \\ &= \sqrt{x^2-4x} + 2 \ln |x-2 + \sqrt{x^2-4x}| + C\end{aligned}$$

$$\begin{aligned}12. \int_0^{\pi/2} \cos^5 x dx &\stackrel{74}{=} \frac{1}{5} [\cos^4 x \sin x]_0^{\pi/2} + \frac{4}{5} \int_0^{\pi/2} \cos^3 x dx \\ &\stackrel{68}{=} 0 + \frac{4}{5} \left[\frac{1}{3} (2 + \cos^2 x) \sin x \right]_0^{\pi/2} = \frac{4}{15} (2-0) = \frac{8}{15}\end{aligned}$$

13. Using Formula 95 with $n = 2$,

$$\begin{aligned}\int x^2 \tan^{-1} x dx &= \frac{1}{3} \left[x^3 \tan^{-1} x - \int \frac{x^3 dx}{1+x^2} \right] \\ &= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{x^2+1} \right) dx \\ &= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \frac{x^2}{2} + \frac{1}{6} \int \frac{du}{u} \quad \left[\begin{array}{l} u = x^2+1, \\ du = 2x dx \end{array} \right] \\ &= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2) + C\end{aligned}$$

$$\begin{aligned} 14. \text{ Volume} &= \int_0^1 \frac{2\pi x}{(1+5x)^2} dx \\ &\stackrel{51}{=} 2\pi \left[\frac{1}{25(1+5x)} + \frac{1}{25} \ln |1+5x| \right]_0^1 \\ &= \frac{2\pi}{25} \left(\frac{1}{6} + \ln 6 - 1 - \ln 1 \right) \\ &= \frac{2\pi}{25} \left(\ln 6 - \frac{5}{6} \right) \end{aligned}$$